Elektrijada 2009.

Problems for the competition in the area of
CONTROL SYSTEMS

1. The system open-loop transfer function is $W(s) = \dfrac{10}{s(s+6)}$.

a) Obtain the steady-state error of the closed-loop system for the reference signal

$$r(t) = 2\left[h(t-4) - 3h(t-1)h(t-5)\right].$$

b) Obtain the following characteristics of the system:

- Phase margin and
- Closed-loop bandwidth.

c) Obtain and sketch the response of the closed-loop system when unity step reference is applied. Obtain the value of the overshoot and the time when the response reaches 50% of its steady-state value.

d) If the sensor with pure time delay τ is introduced in the closed loop, obtain the maximal value τ_{max} which will not destabilize the closed-loop system.

e) What should be the value τ of the pure transport delay to perform simultaneous decrease of the phase margin to the half (of original value without transport delay) and doubling of the unity-gain crossing frequency (comparing to the original value without transport delay)?

Remark: Unity step (Heaviside) signal is denoted by $h(t)$.

2. Model of the system is given in the state-space:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad y = \mathbf{C}\mathbf{x}, \quad \mathbf{A} = \begin{bmatrix} a & 1 \\ -1 & -2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

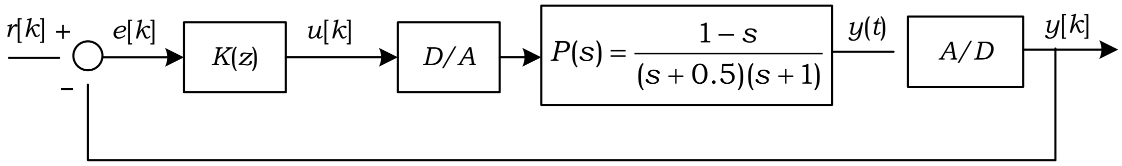
and it has unknown initial conditions $\mathbf{x}(0)$. When the control signal $u(t) = 3\delta(t)$ is applied to the system, it was found that the system response is identically equal to zero.

a) Obtain the initial conditions $\mathbf{x}(0)$, depending on the parameter a.

b) On the basis of previous analysis in a), comment the opservability of the system states.

Remark: Dirac impulse signal is denoted by $\delta(t)$.

3. Control system is depicted in the figure:



Applied combination of digital/analog D/A and analog/digital A/D conversion is modeled as zero order hold circuit and ideal sampling. Sampling period is $T = \ln 2$ sec.

a) Obtain the transfer function $P(z) = Y(z)/U(z)$.

b) Design the controller $K(z)$ that will make the response of the closed-loop system:

$$y(t) = \left(1 - e^{-(t-\tau)/T_d}\right) h(t-\tau), \quad T_d = 2T, \quad \tau = 3T,$$

when unity step reference $r[k] = h[k]$ is applied.

c) Obtain the steady-state value of the control signal $u[k \to \infty]$, which is produced by the controller $K(z)$ designed in b), when unity step reference $r[k] = h[k]$ is applied.

Remark: Unity step (Heaviside) signal is denoted by: $h(t)$ for continuos signal and $h[k]$ for discrete signal.