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Problems for the competition in the area of CONTROL SYSTEMS

1. Digital system, with input $u[k]$ and output y[k], is defined by the set of equations:

$$x_1[k+1] = x_2[k] + 2u_1[k]$$
$$x_2[k+1] = 3x_1[k] + 4u_2[k]$$
$$y[k] = x_1[k] + x_2[k]$$

a) Form the system model in the space of states $x[k] = \left[x_1[k] \;\; x_2[k]\right]^T$.

b) Discuss controlability and opservability of the model states.

c) The goal is to take the system from the initial state $x[0] = [-1 \;\; 1]^T$ to the desired state $x^* = [1 \;\; -1]^T$. If possible, determine the minimal number of control signal samples N and values of the control signal $u[0]$, …, $u[N\text{-}1]$, which will make the desired transition. If it is not possible, give the reason.

d) Measurements of the system output $y[0] = 2$, $y[1] = 3$ are given. If possible, determine initial state $x[0] = \left[x_1[0] \;\; x_2[0]\right]^T$ of the system (there is no control signal applied). If it is not possible, give the reason.

2. For the system $G(s) = \dfrac{2}{\left(s/10+1\right)^2}$

a) Design PI controller $K_{PI}(s) = \dfrac{K(s/\omega_n + 1)}{s}$, to make regulated system have desired characteristics:

- steady-state tracking error for the ramp reference not more than 10%,
- phase margin $\Phi_{pf} \geq 45°$,

b) Choose adequate sampling time T_s for PI controller discretization and explain your choice.

c) Discretize controller $K_{PI}(s)$ with Tustin (bilinear) transform and represent discrete equivalent $K_{PI}(z)$ by set of diference and algebraic equation, suitable for implementation of control algorithmic on digital computer.

3. For the open-loop system $W(s) = \dfrac{2}{s(s+2)}$:

a) Obtain closed-loop transfer function $T(s)$ and find value and time moment of the overshoot in response to unity-step reference.

b) Find the bandwidth of the closed-loop system.

c) Obtain closed-loop discrete transfer function $T(z)$, using impulse invariance method with sampling time $T_s = 1$ sec.

d) Obtain the response of discrete equivalent $T(z)$ to unity step reference, and find the time moment kT when the difference of the original continuous system and his discrete equivalent is the biggest.